Analytical 3D Hydrodynamical Analysis of Spurious Compressional Wave Excitation by Microacoustic TSM Liquid Sensors

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Piezoelectric thickness-shear-mode (TSM) resonators are well-established as viscosity and chemical liquid sensors. When immersed in the sample liquid, the resonator excites a strongly damped shear-polarized wave as well as spurious compressional waves in the liquid. The latter are scarcely damped, which can lead to disturbing interferences if they are reflected by obstacles close to the sensor. In order to analyze the spurious compressional wave excitation due to the non-uniform shear displacement across the resonator/liquid-interface, we developed a three-dimensional mathematical model utilizing solutions of the linearized Navier-Stokes equations in the spatial Fourier-domain, which govern the acoustic field in the liquid. We discuss the resulting solutions and illustrate these results by considering practical examples.

Basic Sensor Concept

A typical TSM-resonator for viscosity sensing applications is made of a thin AT-cut quartz disk, which is plated with metal electrodes on both sides of the disk (see Fig. 1). When applying an electric alternating voltage to the electrodes, the resonator starts to vibrate in thickness-shear-mode. The adjacent liquid is entrained by the shear movement of the immersed TSM-resonator, which leads to a change of the resonance frequency and the damping compared to the vibrating resonator in air. By detecting this shift, the viscosity-density product of the liquid can be determined [1], [2].

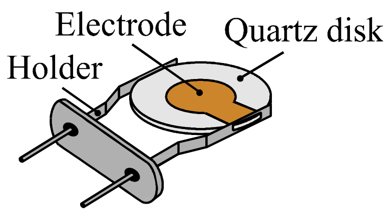


Fig. 1: Typical design of a TSM-resonator for viscosity sensing applications

In the ideal case, a pure and uniform shear displacement across the resonator surface leads to excitation of an evanescent shear wave in the adjacent viscous liquid. However, in the real case, spurious compressional waves are also excited because of

- small out-of-plane displacements due to mode conversion (at clamping points, electrode edges, ...),
- the uncompensated angular momentum of the dominant shear vibration of the quartz disk [3].

Method and Theory

This contribution deals with a generalization of a recently developed two-dimensional model [1]. We decouple the piezoelectric disk and the fluid region (see Fig. 2 (a)) by considering only the liquid and replace the resonator by impressing a non-uniform Gaussian-shaped shear displacement at the interface as it has been observed in practical measurements [1], [2]. Then, the linearized Navier-Stokes equation in the liquid

$$\nabla^2 \underline{\mathbf{u}} + \alpha \nabla(\nabla \cdot \underline{\mathbf{u}}) + \beta \underline{\mathbf{u}} = \mathbf{0},$$

where α and β are fluid parameters [4] and $\underline{\mathbf{u}}(x,y,z)$ is the displacement vector in the liquid, is solved by using a spectral method. The displacement components $\underline{u}_i(x,y,z)$ in the liquid ($i \in \{x,y,z\}$) are represented in terms of compact Fourier double-integrals

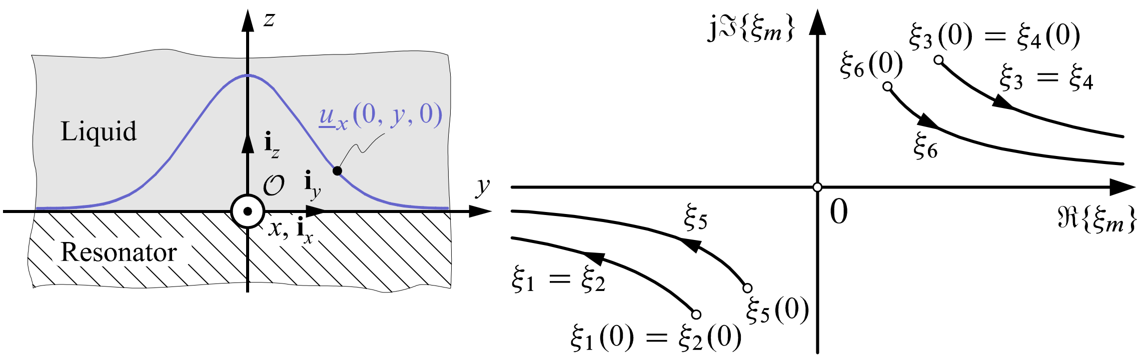
$$\underline{u}_i(x,y,z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \underline{U}_i(k_x,k_y;z) \exp[-\mathrm{j}(k_x x + k_y y)] \, \mathrm{d}k_x \, \mathrm{d}k_y \, .$$

Here, k_x and k_y are the lateral wave numbers and $\underline{U}_i(k_x,k_y;z)$ is the general solution ($i \in \{x,y,z\}$) in the spectral domain, which reads

$$\underline{U}_x\left(k_x,k_y;z\right) = \underline{U}_{x0} \left[\frac{\left(k_y^2 - \xi_1\xi_5\right)\exp\left(\xi_1 z\right)}{k_x^2 + k_y^2 - \xi_1\xi_5} + \frac{k_x^2 \exp\left(\xi_5 z\right)}{k_x^2 + k_y^2 - \xi_1\xi_5} \right],$$

$$\underline{U}_y\left(k_x,k_y;z\right) = -\underline{U}_{x0} \frac{k_x k_y \left[\exp\left(\xi_1 z\right) - \exp\left(\xi_5 z\right)\right]}{k_x^2 + k_y^2 - \xi_1\xi_5},$$

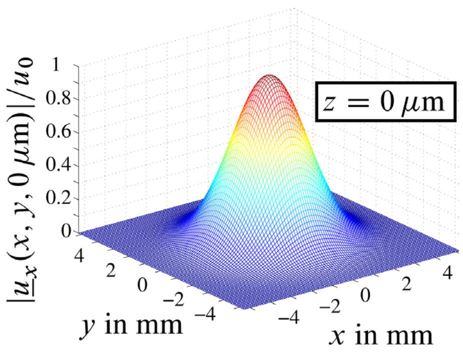
$$\underline{U}_z\left(k_x,k_y;z\right) = -\underline{U}_{x0} \frac{\mathrm{j} k_x \xi_5 \left[\exp\left(\xi_1 z\right) - \exp\left(\xi_5 z\right)\right]}{k_x^2 + k_y^2 - \xi_1\xi_5}.$$



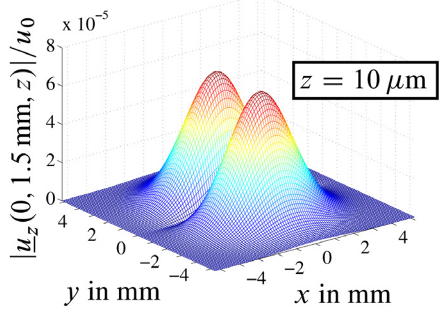
(a) (b)

Fig. 2: (a) Half space model to analyze the wave excitation mechanism of a vibrating TSM-resonator immersed into a liquid. The area $z > 0$ is filled with a linear viscous and adiabatic compressible liquid. The resonator is replaced by an appropriate boundary condition at $z = 0$. (b) Schematic locus of the eigenvalues ξ_m as function of $k = (k_x^2 + k_y^2)^{1/2}$. Note that the real parts of ξ_1, ξ_2, and ξ_5 are negative for any arbitrary value of k, while the real parts of ξ_3, ξ_4, and ξ_6 are always positive. For common liquids is $|\Re\{\xi_1\}| \ll |\Re\{\xi_5\}|$ resulting in a much lower damping for compressional waves.

$\underline{U}_{x0}$ is the 2D-Fourier transform of the boundary condition at $z=0$. $\xi_1=(k_x^2+k_y^2-\beta)^{1/2}$ and $\xi_5=(k_x^2+k_y^2-\beta/(1+\alpha))^{1/2}$ are the eigenvalues associated to the problem (see Fig. 2 (b)). The real parts of ξ_1 and ξ_5 are related to spatial damping in z-direction, while the imaginary parts correspond to a wave number in z-direction. For small k-values, ξ_5 corresponds to a dominantly longitudinal mode, whereas ξ_1 is related to a dominantly shear polarized mode.



(a)



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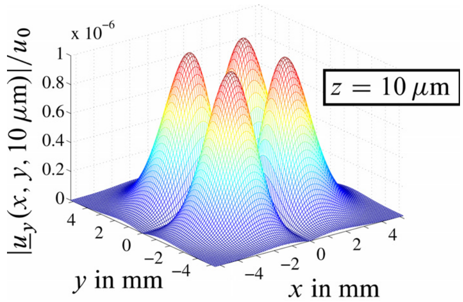
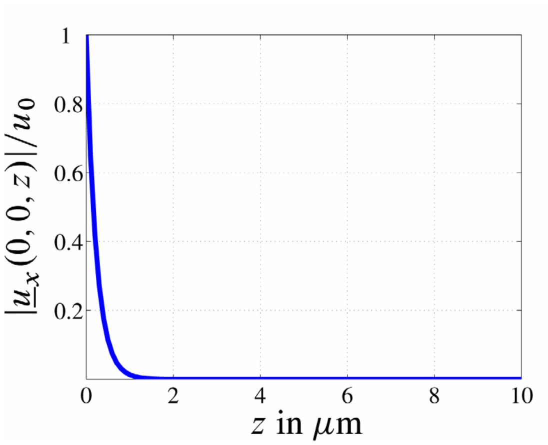


(c)

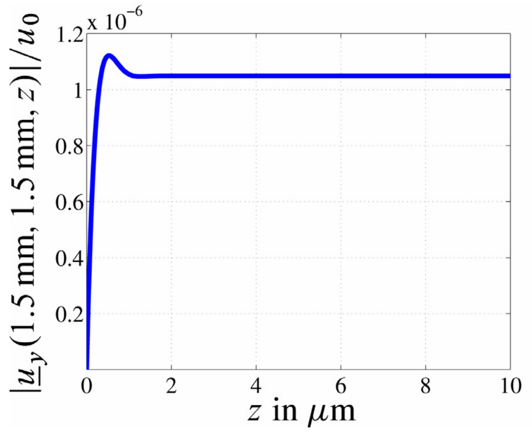
Fig. 3: (a) Normalized shear displacement $|\underline{u}_x(x,y,0)/u_0|$ at the resonator surface $z=0$ as prescribed by the boundary condition. (b) Normal displacement $|\underline{u}_z(x,y,10\,\mu\text{m})/u_0|$ at $z=10\,\mu\text{m}$ above the resonator surface. Two major centers of scarcely damped compressional waves are built up above the resonator. (c) Secondary shear component $|\underline{u}_y(x,y,10\,\mu\text{m})/u_0|$ at $z=10\,\mu\text{m}$ above the resonator surface. This component shows four major centers and is a by-product of the compressional wave beams.

Results

We consider a frequency of 6 MHz and water as sample liquid. As a result, a dominantly shear polarized wave $\underline{u}_x$ is excited (see Fig. 3 (a)), which is highly damped in z-direction (see Fig. 4 (a)). Simultaneously, two major beams (see Fig. 3 (b)) of scarcely damped compressional waves (see Fig. 4 (b)), represented by normal displacements $\underline{u}_z$, are built up with increasing z. Due to their finite lateral extension, these pressure waves cause secondary shear components in x- and y-directions with amplitudes being typically two orders of magnitude smaller than the associated normal (see Fig. 3 (c)) components $\underline{u}_z$. At this height above the surface, the related secondary amplitudes in x-direction are still overshadowed by the primary Gaussian shear displacements. Their damping behavior is similar to one of the compressional waves (see Fig. 4 (c)).



(a)



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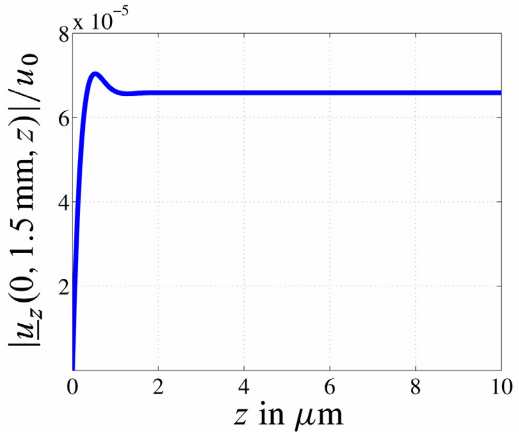

(c)

Fig. 4: Comparison of the major peaks of the displacement components with increasing distance z from the resonator surface.

References

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[4] R. Beigelbeck and B. Jakoby: "A Two-Dimensional Analysis of Spurious Compressional Wave Excitation by Thickness-Shear-Mode Resonators", Journal of Applied Physics, 95(9):4989, 2004.