Coupled Surface States in Thin, Frequency Dependent Layers

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In photonic crystals made of materials with a frequency dependent dielectric function surface plasmon modes are extending over some adjacent layers and it turns out, that the dispersion can be tailored by coupling surface plasmons of adjacent layers. To give a detailed study we start investigating surface plasmons on a single metallic sheet. The mode dispersion is not bijective any more (i.e. is bent back) due to the coupling of surface plasmons and radiative modes, and the group velocity diverges and is even negative. This effect is still physical as the absorption of the metal influences the properties of the surface plasmons and yields an inequality between group and energy velocity. Next to these effects we investigate the propagation distance of the surface states and optimize them by varying the thickness of the layers. Finally we calculate the dispersion for two adjacent layers to see possible coupling effects.

Introduction

Photonic crystals (PC) have been investigated for two decades theoretically and experientially as they allow to model the dispersion of this artificial material according to the needs and interests [1].

A very efficient approach was presented by Kuzmiak et al. by using metal in one and two-dimensional PCs [2]. In the one-dimensional case the large imaginary part (and via Kramers-Kronig relations negative real part) of the dielectric function below the plasma frequency leads to the formation of additional band gaps which comes clearly from absorption. This absorption band gap can be controlled by the thickness of the metallic layers (i.e. the filling factor) and the penetration depth is much smaller in this absorption band gap than in the band gaps arising from the structure and dielectric difference of the two composites. Further, it turns out (for example by studying a polaritonic PC where one material has a Lorentzian dielectric function) that these absorption band gaps are omnidirectional for both polarizations and angles of incidence [3].

In material with a negative dielectric function also surface plasmons can be excited [4]. These modes lie below the light line and are propagating almost parallel to the surface of a metal. Surface plasmons have a huge potential in applications as their coupling to free space is forbidden without any structural changes (like a grating). They can be guided, e.g., by a bent metallic ridge.

In this work we study surface plasmons on metallic interfaces as it was done in [5] (without damping) and [6] (for one silver layer) for some parallel layers and compare the resulting solutions with the dispersion of the surface plasmons in a metallic PC. We are also interested in the coupling effects of different modes in different frequency regions which results in a negative and back bent dispersion. Further we investigate the propagation length of surface plasmons on a single film with respect to the layer thickness.

The group velocity becomes negative and infinite. We study the expressions of the group and energy velocity and show that in absorbative systems the group velocity loses its meaning.

Surface Plasmons in Thin Films

Surface plasmons on a single metal-air interface are described by

$k_y = \omega(\varepsilon_b\varepsilon(\omega)/(\varepsilon_b\ \varepsilon(\omega))^{1/2}$.

In this equation k_y is the in-plane component of the wave vector, ω the frequency and ε_b and $\varepsilon(\omega)$the dielectric function of the background and the metal, respectively. In our contribution we will follow literature and use the Drude model to describe the metal. By doing so we obtain different modes, namely bound surface modes below $\omega_a := \omega_p/\sqrt{2}$, quasi bound modes between ω_a and the plasma frequency ω_p and radiative modes above ω_p (see [6]). The quasi bound modes result from the coupling of surface modes with the radiative ones and are only obtained by taking the imaginary part of the Drude dielectric function into account. This coupling results in a negative dispersion which is therefore connected to a large damping via the Kramers-Kronig relations. Therefore these quasi bound modes only propagate small distances which can be optimized by changing the thickness of the metallic film.

A thin metallic film shows two solutions (symmetric and antisymmetric) which are obtained from an implicit equation:

$\varepsilon(\omega)k_b/\varepsilon_b k_m = -\tanh(k_m d/2) = -\operatorname{ctanh}(k_m d/2)$

In this equation k_m and k_b indicate the normal component of the wave vector in the metal or the background, respectively. With the help of a Nelder-Mead minimization algorithm [8] one obtains both solutions for arbitrary thickness and damping of the metallic film. The solutions were often discussed in literature [5], [7] and therefore we mention that one solution is strongly damped whereas the other one shows small damping and is quite similar to the mode on a single interface except that the asymptotic frequency ω_a shifts towards ω_p by decreasing the thickness of the film.

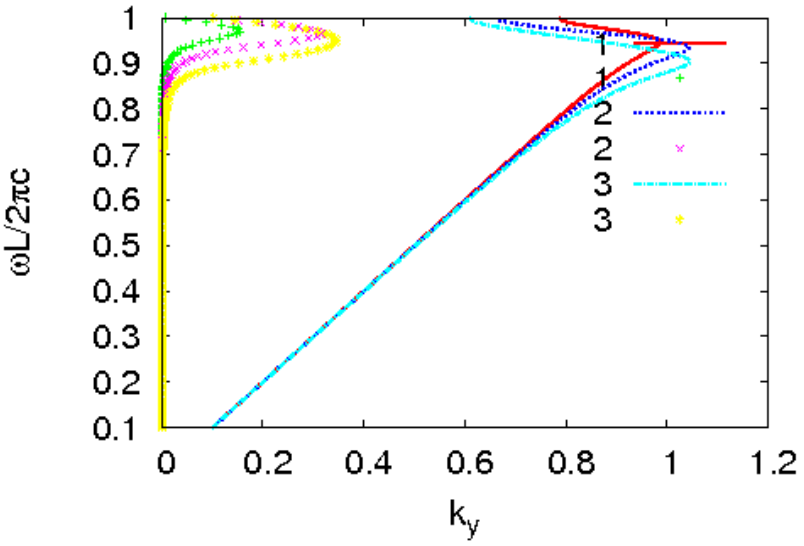

Fig. 1: The hardly damped solutions for one (red), two (blue) and three layers (turquoise). The imaginary part of the dispersion is displayed in green, magenta and yellow for one, two and three adjacent sheets, respectively. The thickness of the metallic film is always kept constant.

Adding a second metallic sheet parallel to the first one allows to study the coupling of surface plasmons. The four solutions can again be obtained from an implicit equation

$A^4\text{-}V\,A^2 + e^{-4kmd}$

with $A = (k_b\,\varepsilon(\omega) - k_m\varepsilon_b)/(k_b\,\varepsilon(\omega) + k_m\varepsilon_b)$ and $V = 2e^{-2kmd} + e^{-2kbL}(1-e^{-2kmd})^2$. We solve this equation again by the Nelder-Mead algorithm. In Fig. 1 the thickness of the layers is $d = 0.1\ \omega_p$ and the distance between the sheets is $L = \lambda_P$ (which is the free space wavelength of the plasma frequency).

One can see that the damping increases for an increasing number of layers (green, pink and yellow dots) and the asymptotic frequency in the real part of the dispersion goes towards $\omega_p/\sqrt{2}$ (red, blue, cyan curves). For each additional layer more the number of solutions increases by a factor of two which should lead to an infinite number of surface plasmon solutions in the metallic PC. The surface modes in the PC are obtained by determining the solutions for a fixed value of the Bloch wave vector (in [3] the solution for $k_{Bloch} = 0$ is presented which is almost similar to the solution on a single interface). In [5] it is suggested that the solutions for an infinite amount of parallel metallic sheets the solutions should lie in an area which is bounded by $k_{Bloch} = 0$ and $k_{Bloch} = \pi/L$.

Group Velocity

In the previous section we saw that the coupling of the bounded and the radiative mode yield a negative, back bent dispersion. The group velocity, which is defined by $v_g = d\omega/dk$, is therefore negative and at certain points infinite which is of course unphysical. Many authors have shown that in absorbing systems such superluminal effects are obtained (e.g. see [9]).

We show here that by studying Poyntings theorem S = E x H and the resulting continuity equation

$-\mathrm{div}(S) = d/dt\ u + j\,E$

one finds that the energy velocity is not equal to the group velocity (which is unphysical at certain points) as the energy density reads [10] in first approximation

$u \sim d\,(\varepsilon(\omega)\,\omega)/d(\omega).$

Due to the imaginary part of the dielectric function of the metal this inequality arises and additional terms contribute to the expression for the energy velocity. Therefore the group velocity looses its meaning for absorbing materials.

Conclusion

In our contribution we have studied the negative dispersion of surface plasmons which arises from the coupling of bound and radiative modes. The negative dispersion results in unphysical values for the group velocity. We have pointed out that the group velocity becomes meaningless in absorbing systems as additional terms modify the expression for the energy velocity. Further we have investigated how the dispersion of surface plasmons changes for parallel metallic layers and have compared these modes with surface plasmons in a metallic photonic crystal.

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