Measurement of Liquid Properties with Resonant Cantilevers

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Liquid viscosity and density sensors are essential devices in online process or condition monitoring. Microacoustic sensors combine advantages such as small size, low cost, and absence of macroscopically moving parts. However, these devices feature measurement at high shear rates and, therefore, the results may diverge from those obtained with traditional viscometers. In our contribution we investigate a PZT driven bending actuator and a micromachined doubly clamped bridge device for use as viscosity and density sensors. The presented models allow the description of the sensor's interaction with the surrounding liquid.

Introduction

In many applications like online process or condition monitoring liquid viscosity and density are of high relevance. Microacoustic sensors such as quartz crystal resonators and SAW devices have proved particularly useful alternatives to traditional viscometers [1]. However, these devices basically measure liquid viscosity at high shear rates. The results of microacoustic measurements are thus often not comparable to the readout of conventional viscosity measurement, e.g., based on the Ubbelohde method or rotational viscometers [2]. In contrast, resonating cantilevers allow for measurements at lower frequencies and higher shear amplitudes, making the measuring results more comparable to laboratory methods [3]. Furthermore they allow the determination of liquid viscosity and density separately [4].

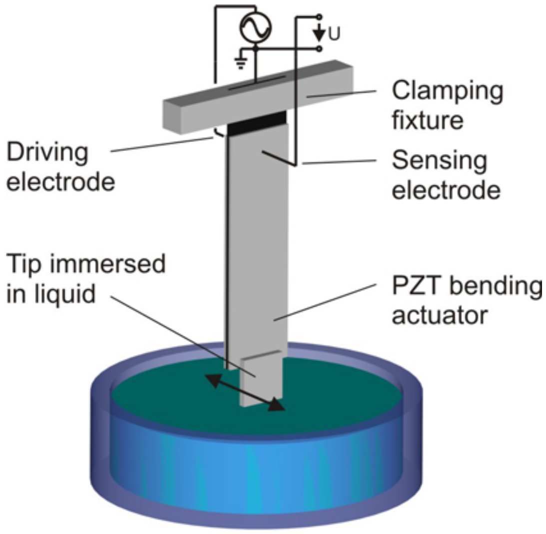
In our contribution, we investigate two different types of resonant cantilever sensors: A PZT (lead zirconate titanate) driven bimorph bending actuator and a micromachined sensor utilizing a doubly clamped vibrating beam.

PZT Cantilever Sensor

Sensor Fabrication

Commercially available PZT bimorph bending actuators consist of two piezoelectric PZT layers polarized in the thickness direction and a carbon fiber substrate. Electrodes on both sides of the piezoelectric layers enable the excitation of the actuator. A voltage applied to the electrodes causes transversal contraction in the piezoelectric layer but not in the substrate, and, therefore, leads to flexural mode vibrations of the beam.

The bending actuators used in this work were supplied by Argillon GmbH, Redwitz, Germany. The center electrode is used as ground electrode (Fig. 1(a)), whereas a sinusoidal voltage is applied to the driving electrode to excite bending vibrations. The actual beam deflection is determined by measuring the voltage at the sensing electrode. The phase shift between driving voltage and sensor voltage is measured by means of a lock-in amplifier (Stanford Research SR830). In our setup (Fig. 1(a)), the bending actuator is clamped at one end, whereas different tips of well-defined cross-sections have been attached to the free end of the cantilever (fig. 1b). The clamping fixture is mounted on a rigid frame allowing for vertical positioning of the sensor and preventing vibrations of the whole setup.



	Tip	Width [mm]	Thickness [mm]
A	Silicon	7	0.5
B	Silicon	5	0.5
C	Silicon	2	0.5
D	terminal pin, soldered on	0.7	0.7

(a) (b)

Fig. 1: (a) Measurement setup; (b) Tip geometries investigated in this work.

Characterization of the PZT Cantilevers

From the cantilever's frequency response we obtained the resonance frequency f_n of the considered vibration mode n and the damping factor D. These parameters are influenced by the liquid's viscosity η and density ρ and the cross-section of the immersed tip. In [5] the relationships between f_n, D and η, ρ were modeled by approximating the oscillating cantilever by an oscillating sphere immersed in a liquid. The force F acting on the said sphere is given by [6]

$$F = \underbrace{6\pi\eta\left(1+\frac{R}{\delta}\right)}_{b_i} u + \underbrace{3\pi R^2\sqrt{\frac{2\eta\rho}{\omega}}\left(1-\frac{2R}{9\delta}\right)}_{M_i}\frac{du}{dt}, \quad \delta = \sqrt{\frac{2\eta}{\omega\rho}}, \tag{1}$$

where R is the sphere radius, ω the angular frequency, and δ the depth of penetration of the acoustic wave. The terms b_i and M_i represent the additional damping of the cantilever and the added mass due to the surrounding liquid. Considering the bending actuator as an oscillator immersed in a liquid and driven by a harmonic force, the differential equation for the motion u in axial direction is

$$(M_e + M_i)\frac{d^2u}{dt^2} + (b_e + b_i)\frac{du}{dt} + Ku = F_0 e^{-j\omega t}, \tag{2}$$

where M_e and b_e are the effective mass and the intrinsic damping of the cantilever, K is the spring constant and F_0 and ω are the driving force's amplitude and angular frequency. Based on the oscillating sphere model, a more generalized model for the relationships between $\omega_n = 2\pi f_n$, D and η, ρ was deduced, given by

$$\omega_n = \omega_{n,AIR}\sqrt{\frac{1}{1+c_1\rho + c_2\sqrt{\eta\rho}}} \quad \text{and} \quad \frac{D}{\omega_n^{\;2}} = \frac{D_{AIR}}{\omega_{AIR}^{\;2}}\left(1+c_3\eta + c_4\sqrt{\eta\rho}\right). \tag{3}$$

The model parameters c_1, c_2, c_3, and c_4 were obtained by curve fitting to the equations to the measured data. Figure 2 elucidates that the resonance frequency is dominantly influenced by the liquid's density, whereas the damping is mainly determined by the viscosity. However, there is a "spread" in these straightforward bilateral relations which indicates a cross-sensitivity to the respective other liquid parameter and is properly described by the model (3).

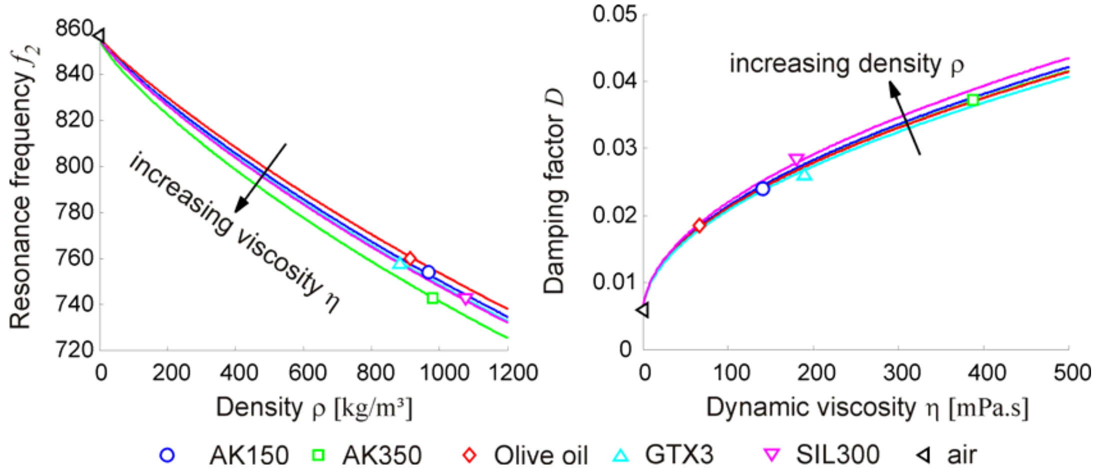


Fig. 2: Measured values (markers) and fitted curves for the respective fluids. These measurements have been obtained with tip A, dipping depth 4 mm.

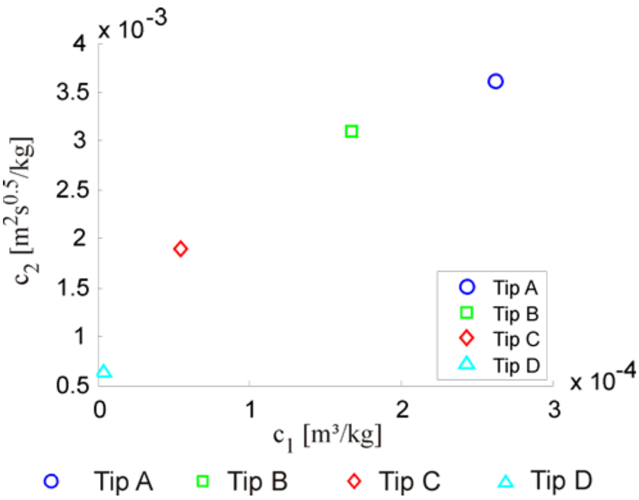


Fig. 3: Fit parameters c_1 and c_2 (associated with $f_n(\eta, \rho)$) for the different tip geometries (Fig. 1(b)).

Figure 3 depicts the dependence of the fit parameters c_1 and c_2 on the kind of tip used. It turns out that the sensitivity of the resonance frequency to the density (determined by c_1) can be steadily increased by increasing the tip width, whereas its sensitivity to the viscosity-density product tends to saturate for increasing widths. This can be explained by the fact that larger amounts of liquid must be moved by the oscillating tip with increasing tip width, whereas the influence of the viscosity is concentrated to the edges of the tip.

Micromachined Doubly Clamped Bridge

Motivated by the applicability for viscosity and density measurement of the PZT cantilevers, effort is being made to miniaturize the sensor with the aim of a sensing device in silicon technology. A precise modeling of the solid-liquid interaction and the measurement of the frequency response enables the measurement of the density and the rheological behavior of liquids [4] in a way comparable to the PZT cantilevers.

A SEM micrograph of a prototype which consists of a doubly clamped beam structure carrying a conductive path is shown in Fig. 4. The beam structure is exposed to a permanent magnetic field and a sinusoidal current through the conductor results in a sinusoidal Lorentz force causing the time-harmonic vibration.

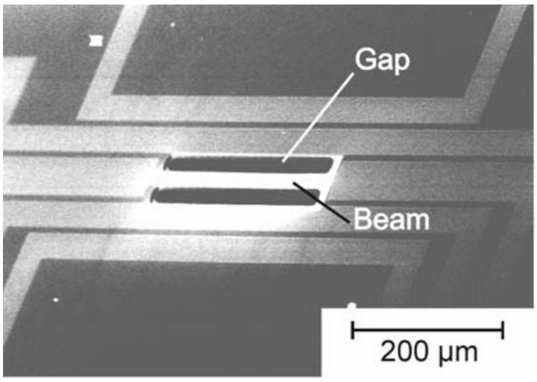


Fig. 4: SEM micrograph of the micromachined sensor element. The cross-sectional dimensions of the Silicon nitride beam are 40x1.3 µm; beams of different lengths ranging from 240 µm to 720 µm were fabricated.

The mechanical deformation (of pure transverse vibration modes) is described by the Bernoulli-Euler beam equation considering an intrinsic normal force N, an additional mass m_f and a viscous damping parameter α both resulting from the flow field around the beam:

$$EJ\frac{\partial^4 w}{\partial x^4} - N\frac{\partial^2 w}{\partial x^2} + \left(\rho A + m_f\right)\frac{\partial^2 w}{\partial t^2} + \alpha\frac{\partial w}{\partial t} = q(x,t)$$

where w is the deflection, E the Young's modulus, J the geometrical inertia, ρ the mass density of the beam material, and x the spatial coordinate along the beam, $q(x,t)$ is the driving force per unit length.

The additional mass per unit length m_f and the viscous damping coefficient α were calculated from a semi-numerical method based on Green's function in the spatial spectral domain. The simulation of the two-dimensional flow field around the rectangular cross-section yields the dependency of the resistance force on the liquid's mass density ρ_f and viscosity μ shown in Fig. 5.

The readout of the vibration amplitude can be carried out optically, with the integration of piezoelectric materials, or as done here, by measuring the impedance of the excitation circuit over the appropriate frequency range. The electrical equivalent circuit of the vibrating beam in an external magnetic field is depicted in Fig. 6.

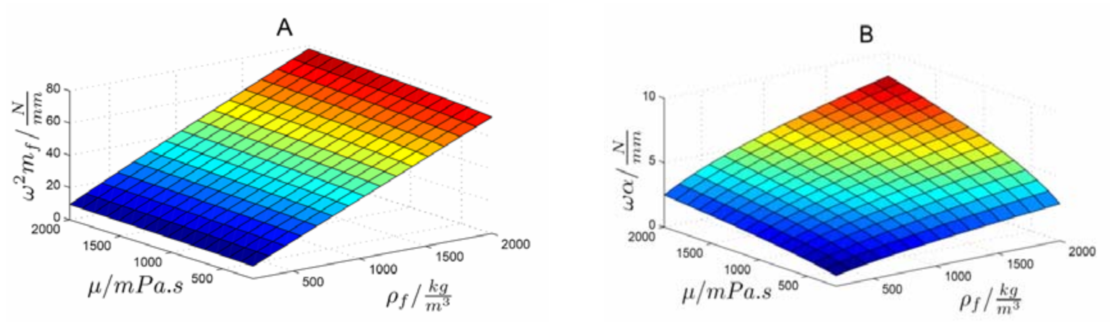


Fig. 5: Results of the semi-numerical analysis of the flow field around the rectangular cross-section of the beam. A: Dependency of the resistance force (per unit length) due to the added mass on both viscosity μ and liquid's density ρ_f. B: Force due to the viscous damping parameter α.

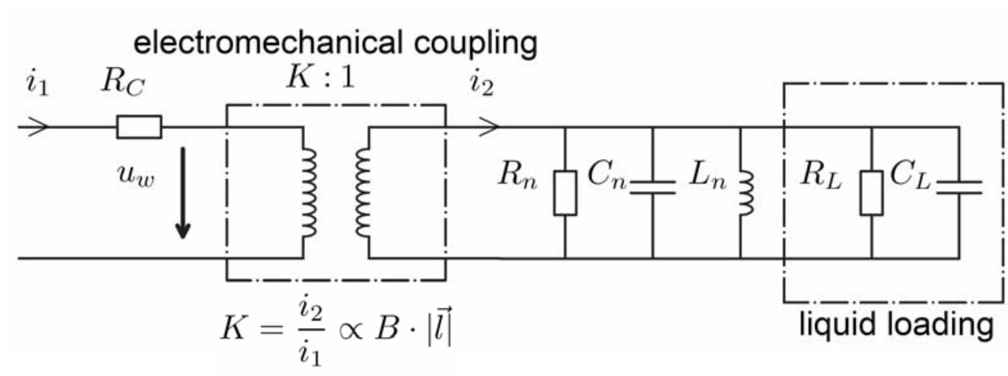


Fig. 6: Electrical equivalent circuit of the vibrating structure. R_C represents the resistance of the excitation path. The parallel resonance circuit on the secondary side of the transformer represents the lumped element approximation of the mechanical system, valid for one specific mode of vibration, and the fluid dependent elements R_L and C_L.

Using the equivalent circuit model it is possible to determine the liquid loading and thus the fluid properties from a measurement of the impedance spectrogram:

$$Z(\omega) = R_C + K^2 \frac{1}{\frac{1}{R_n} + \frac{1}{R_L} + j\omega(C_n + C_L) + \frac{1}{j\omega L_n}}$$

where R_n, C_n, and L_n are the resonance parameters of mode n in air and R_L and C_L represent the change of the behavior when the structure is immersed in a liquid due to additional damping and additional mass respectively.

Conclusion

In the paper we have investigated two different types of resonant cantilevers for the measurement of liquid properties. The resonance frequency as well as the damping of the cantilevers are influenced by the surrounding liquids. The relationships between the cantilever's frequency response and the liquid parameters were described by models, allowing for separate determination of liquid viscosity and density.

Acknowledgements

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